

Neutrino Constraints on Spontaneous Lorentz Violation

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Abstract

We study the effect of spontaneous Lorentz violation on neutrinos. We consider two kinds of effects: static effects, where the neutrino acquires a Lorentz-violating dispersion relation, and dynamic effects, which arise from the interactions of the neutrino with the Goldstone boson of spontaneous Lorentz violation. Static effects are well detailed in the literature. Here, special emphasis is given to the novel dynamic effect of Goldstone-Čerenkov radiation, where neutrinos moving with respect to a preferred rest frame can spontaneously emit Goldstone bosons. We calculate the observable consequences of this process and use them to derive experimental bounds from SN1987A and the CMBR. The bounds derived from dynamic effects are complementary to — and in many cases much stronger than — those obtained from static effects.

I. INTRODUCTION

Neutrinos provide an interesting laboratory for studying possible violations of Lorentz invariance. First, neutrino oscillation experiments indicate that neutrino mass differences are much smaller than 1 eV. Thus, neutrinos should be sensitive to small deviations from relativistic energy-momentum relations. Second, because neutrinos are so weakly interacting, they are sensitive to physics over very long times and distances. Neutrinos from distant astrophysical sources like supernovae can probe very small effects that accumulate throughout the time of travel. Finally, neutrinos constitute a significant amount of the energy of the universe during the time the Cosmic Microwave Background Radiation (CMBR) was generated. During this epoch, neutrinos are decoupled from the baryon-photon plasma and free-stream from over- to under-dense regions. Any deviation from this picture can affect the CMBR signals we observe today.

While the specific origin of Lorentz violation (if any) is unknown, there are some model-independent statements we can make.¹ If general relativity is the correct description of gravity up to the Planck scale, then Lorentz violation must be spontaneous. The reason is that any operator that breaks Lorentz invariance necessarily violates space-time diffeomorphisms, which is the gauge symmetry of gravity. As with all gauge symmetries, diffeomorphisms (and hence Lorentz invariance) can only be broken spontaneously. A spontaneously broken symmetry implies the existence of a Goldstone boson, though in a Lorentz-invariant context, this Goldstone boson would be “eaten” by the gauge field and all physical polarizations would become massive. However, if we break both diffeomorphisms and Lorentz invariance, the physical Goldstone boson can be exactly massless and lead to novel interactions in the infrared. At minimum, the Goldstone boson of spontaneous Lorentz violation will mix with gravity, which may or may not lead to measurable modifications of gravitational physics.

In this paper, we focus on possible direct couplings between neutrinos and the Lorentz-violating sector which survive in the $M_{\text{Pl}} \rightarrow \infty$ limit. In order to do explicit calculations, we have to make some assumptions about the Lorentz-violating sector. We consider two models where a scalar field acquires a Lorentz-violating vacuum expectation value (vev), “ghost condensation” [1] and “gauged ghost condensation” [9]. In addition to providing a systematic way for studying Lorentz violating effects, ghost condensation yields a consistent infrared modification of gravity [1, 10] and has been used as a novel model of inflation [11, 12]. Direct couplings between the ghost condensate and the standard model were considered in [13], where several dynamic phenomena involving the Goldstone boson were identified and studied. We extend this work by investigating how Čerenkov radiation from neutrinos into the Goldstone field can be used to probe spontaneous Lorentz violation.

In general, couplings between any Lorentz-violating sector and neutrinos produce two

¹ There is a growing literature on specific Lorentz-violating modifications of gravity. See, for example [1, 2, 3, 4, 5, 6, 7, 8].

kinds of effects which we categorize as “static” and “dynamic.” The static effects arise from Lorentz-violating vevs, which define a preferred “ether” frame. We choose this frame to be the CMBR rest frame since in the context of ghost condensation, the two frames are aligned due to Hubble friction [1]. The dominant static effect is a modification of the neutrino dispersion relation, which may or may not be CPT-violating. Lorentz-violating static effects in the neutrino sector were studied, for example, in [14, 15, 16, 17, 18, 19], and there is a large literature on the effect of preferred frames on standard model fields [14, 20, 21, 22, 23, 24].

Dynamic effects arise from couplings between neutrinos and the Goldstone boson of spontaneous Lorentz violation. In particular, neutrinos can lose energy while traveling in vacuum as they emit Goldstone-Čerenkov radiation. The parameters that control the interactions with the Goldstone boson are related to those that enter into static effects, thereby providing complementary tests of Lorentz invariance. We emphasize that while the details of the dynamic effects depend on the specific interactions of the Goldstone boson, the existence of a Goldstone boson is robust, so we expect our results to have analogs in any theory where neutrinos are directly coupled to a Lorentz-violating sector.

Spontaneous Lorentz violation is very similar to the well known matter effect in neutrino oscillations [25]. In both cases the neutrino travels in a non-trivial background that breaks Lorentz invariance. For the matter case, the effect is generated by the weak interaction between the neutrinos and the medium they travel through, whereas in ghost condensation, the effect exists in vacuum. In both cases, the Lorentz-violating background generates a static effect, namely an effective “mass” for the neutrino. There is, however, a fundamental difference. For the matter effect the full theory of the weak interaction and the values of its parameters are known. In particular, inelastic interactions between neutrinos and matter (*i.e.* phonon production) are usually negligible. In ghost condensation, on the other hand, dynamic interactions with the Goldstone boson are important for generic values of the parameters in the effective theory. Also, in the case of matter effects, one usually considers only the lowest dimension operators since higher dimension ones are known to be much smaller. In ghost condensation, however, one needs to consider higher dimension operators as well, as the lowest dimension operators may be forbidden (or suppressed) by symmetries of the underlying ultraviolet theory.

By assuming that the Lorentz-violating sector is (gauged) ghost condensation, we are focusing only on the case where Lorentz symmetry is spontaneously broken to rotational symmetry. In the most general case, rotational symmetry can also be broken, yielding additional static effects and presumably new Goldstone bosons. As far as neutrinos are concerned, violations of rotational invariance generate helicity flip tensor operators, which is equivalent to the effect of neutrino traveling in a magnetic field [26]. The implication of such interactions in matter were studied in [27]. For the Lorentz-violating case, such couplings were discussed in [15, 16]. For Dirac neutrinos, these Lorentz-violating tensor operators conserve total lepton number but generate left-right oscillations, such that a left-handed

active neutrino will oscillate into a sterile right-handed neutrino. For Majorana neutrinos, they violate lepton number and must be off-diagonal in flavor space, generating oscillations between neutrinos and anti-neutrinos of different flavors. For the remainder of the paper, we do not consider tensor interactions any further.

In the next section, we first summarize the formalism of (gauged) ghost condensation and show how the ghost condensate couples to neutrinos. We then estimate the sizes of various Lorentz-violating static and dynamic effects on neutrinos, using SN1987A and the CMBR to constrain Goldstone-Čerenkov radiation. We find that the two effects yield complementary bounds on the size of Lorentz-violating operators. The main results are listed in the body of the paper while the details of our calculations are given in the appendices.

II. FORMALISM

A. Ghost Condensation

The basic ingredient of ghost condensation is a scalar field ϕ which acquires a Lorentz-violating vev.² The field ϕ has a wrong sign kinetic term near the origin, but this ghost-like instability is stabilized at a non-zero value of $(\partial\phi)^2$. There is a global shift symmetry acting on ϕ as $\phi \rightarrow \phi + c$, so the leading terms in its effective Lagrangian are

$$\mathcal{L} = \frac{1}{8M^4} \left((\partial_\mu \phi)^2 - M^4 \right)^2 - \frac{\beta}{2M^2} (\partial^\mu \partial_\mu \phi)^2 + \dots, \quad (1)$$

where M defines the characteristic scale of Lorentz violation and naturalness suggests $\beta \sim 1$. Going to a preferred “ether” rest frame, we assume that $\partial_\mu \phi$ acquires a vev in the time direction

$$\langle \partial_\mu \phi \rangle = \delta_\mu^0 M^2, \quad (2)$$

such that Lorentz invariance is broken down to rotational invariance. It is convenient to write

$$\phi \equiv M^2 t + \pi, \quad (3)$$

where π is the physical Goldstone boson of the broken symmetry. Expanding to leading order in the Goldstone energy E_π and momenta k , the Goldstone has the following Lorentz-violating dispersion relation

$$E_\pi^2 = \beta \frac{k^4}{M^2} + O(k^6/M^4). \quad (4)$$

This novel dispersion relation leads to interesting dynamics between neutrinos and the Goldstone. Note that the effective theory for π is applicable only for $k \ll M$, so an unspecified

² In a gravitational context, the low energy physics of ghost condensation is uniquely described by the spontaneous breakdown of space-time diffeomorphisms to spatial diffeomorphisms, whether or not there actually exists a field ϕ that accomplishing this breaking pattern [1, 13].

UV completion is necessary to accurately handle large Goldstone energies. The bound on M from mixing between π and the Newtonian potential were studied in [1], where it was found that

$$M \lesssim 10 \text{ MeV}. \quad (5)$$

Because neutrinos often have energies in excess of 10 MeV, bounds on ghost condensation from energetic neutrinos should be regarded as $\mathcal{O}(1)$ estimates.

B. Gauged Ghost Condensation

A simple modification to ghost condensation is to gauge the $\phi \rightarrow \phi + c$ shift symmetry. The resulting theory of gauged ghost condensation has the nice feature that the scale of spontaneous Lorentz violation M can be taken to be much higher, even well above the electroweak scale, without violating experimental bounds on gravity [9]. Let the gauge transformation act on ϕ and the new $U(1)$ gauge field A_μ as

$$\phi \rightarrow \phi + M\alpha, \quad A_\mu \rightarrow A_\mu - \partial_\mu \alpha, \quad (6)$$

such that the gauge invariant derivative is

$$D_\mu \phi = \partial_\mu \phi + MA_\mu. \quad (7)$$

As in ghost condensation case, we choose $D_\mu \phi$ to acquire a vev in the time direction. Minimally coupling the gauge field to equation (1), the leading effective Lagrangian for ϕ and A_μ is

$$\mathcal{L} = -\frac{1}{4g^2}F_{\mu\nu}^2 + \frac{1}{8M^4} \left((D_\mu \phi)^2 - M^4 \right)^2 - \frac{\beta}{2M^2} (\partial^\mu D_\mu \phi)^2 - \frac{\beta'}{2M^2} (\partial_\mu D_\nu \phi)^2 + \dots, \quad (8)$$

where g is the gauge coupling, and we expect $\beta, \beta' \sim 1$. (The β and β' terms appear since there is an ambiguity in covariantizing the β term in (1).) There is a convenient “unitary” gauge where $\phi \equiv M^2 t$,

$$D_\mu \phi \equiv M^2 \delta_\mu^0 + MA_\mu. \quad (9)$$

In this gauge, the field A_0 has mass gM and can be integrated out, and we can absorb β' into a redefinition of g and β . The remaining three degrees of freedom in A_i are comprised of two transverse and one longitudinal mode. To leading order in k/M and assuming $g \ll 1$, the transverse modes have the Lorentz-invariant dispersion relation $E^2 = k^2$ and are irrelevant to our discussion. The longitudinal mode, however, has the following Lorentz-violating dispersion relation:

$$E_L^2 = \beta \left(g^2 k^2 + (1 - \beta g^2)^2 \frac{k^4}{M^2} \right) + \mathcal{O}(k^6/M^4). \quad (10)$$

In order to safely integrate out A_0 , we have to be in the regime $k \ll gM$. In this limit, the normalized polarization vector and dispersion relation are

$$\epsilon_i^L(k) = gk_i/|k|, \quad E_L^2 = \beta g^2 k^2, \quad (11)$$

and $\sqrt{\beta}g$ is essentially the phase velocity of the Goldstone mode. Note that in order to achieve large values of M without drastically modifying gravity in the infrared, $g \gg M/M_{\text{Pl}}$ [9], which is easily satisfied for any M below the GUT scale. Details of the construction of gauged ghost condensation appear in appendix A.

C. Couplings to Neutrinos

We now consider direct couplings between neutrinos and the Lorentz-violating sector. In the following discussion, we neglect all other standard model fermions, making the implicit assumption that Lorentz-violating couplings to left-handed electrons and neutrinos are not necessarily related by electroweak symmetry. The lowest dimension current involving just left-handed neutrinos is

$$J_\mu = \bar{\nu}_L \gamma^\mu \nu_L, \quad (12)$$

so the leading interaction with the ghost condensate is the dimension five coupling

$$\mathcal{L}_{\text{int}} = \frac{1}{F} J^\mu \partial_\mu \phi, \quad (13)$$

where F sets the mass scale for interaction between neutrinos and the Lorentz-violating sector. Note that if the neutrino were exactly massless, this interaction could be removed by a field redefinition on ν_L because left-handed neutrino number would be conserved. Therefore, any process involving equation (13) must be proportional to the neutrino mass, either a Dirac mass or a Majorana mass. Expanding ϕ around its vev as in equation (3), we have two terms that yield complementary Lorentz-violating effects:

$$\mathcal{L}_{\text{int}} = \frac{M^2}{F} J^0 + \frac{1}{F} J^\mu \partial_\mu \pi. \quad (14)$$

The first term gives rise to well-known static effects that change the dispersion relation of the neutrino. The second term generates dynamic effects between the neutrino and the Goldstone boson which are the focus of this paper. Note that the same coupling F sets the size of the static and dynamic effects, and this is generically true in any theory of spontaneous Lorentz violation [13]. Because Lorentz-invariance is violated, there is no *a priori* reason that $J^0 \partial_0 \pi$ and $J^i \partial_i \pi$ should share the same coupling constant. In the context of ghost condensation, however, the couplings are split by the dimension nine operator

$$\mathcal{L}_{\text{int}} = \frac{C_9}{F^5} J^\mu \partial_\mu \phi \partial_\nu \phi \partial^\nu \phi, \quad (15)$$

where naturalness suggests $C_9 \sim 1$, and therefore we generically expect the splitting to be suppressed. We explicitly checked that for $C_9 = 1$ we can safely neglect any difference between the couplings to the time and spatial components of J^μ .

Next, consider coupling J^μ to the gauged ghost condensate:

$$\mathcal{L}_{\text{int}} = \frac{1}{F} J^\mu D_\mu \phi \quad \Rightarrow \quad \frac{M^2}{F} J^0 + \frac{M}{F} J^i A_i, \quad (16)$$

where we have gone to $\phi \equiv M^2 t$ gauge and integrated out A_0 as in the previous section. If the neutrino is exactly massless, then the static effect generated by the J^0 term can be rotated away by a ν_L field redefinition. However, even if we isolate the A_i longitudinal mode by setting $A_i \sim \partial_i \pi$, the dynamic coupling $J^i A_i$ cannot be removed by a field redefinition. We see that in gauged ghost condensation, dimension five Lorentz violation survives in the $m \rightarrow 0$ limit where m denotes the mass of the neutrino.

Note that the couplings in equations (13) and (16) can be forbidden by a $\phi \rightarrow -\phi$ (and $A_\mu \rightarrow -A_\mu$) symmetry in the fundamental theory above the scale M , so we consider the next most relevant set of operators that would give rise to Lorentz violation even if the dimension five couplings vanished. Furthermore, we consider only operators that survive in the massless neutrino limit. Up to total divergences and terms that can be removed by field redefinitions in the massless limit, the leading operators are the following dimension eight operators

$$\begin{aligned} \mathcal{L}_1 &= \frac{1}{F_1^4} T^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi), \\ \mathcal{L}_2 &= \frac{1}{F_2^4} T_\mu^\mu (\partial_\nu \phi) (\partial^\nu \phi), \\ \mathcal{L}_3 &= \frac{1}{2F_3^4} J^\mu (\partial_\mu \phi) (\partial_\nu \partial^\nu \phi), \end{aligned} \quad (17)$$

where the energy-momentum tensor $T^{\mu\nu}$ for a massless fermion is defined as usual

$$T^{\mu\nu} = \frac{1}{2} \bar{\nu}_L (\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) \nu_L - \eta^{\mu\nu} \bar{\nu}_L \gamma^\rho \partial_\rho \nu_L. \quad (18)$$

Note that unlike the dimension five operator, there is no symmetry that can forbid these dimension eight operators. In terms of the Goldstone, we obtain the following new interactions from \mathcal{L}_1

$$\mathcal{L}_1 = \frac{M^4}{F_1^4} T^{00} + 2 \frac{M^2}{F_1^4} T^{0\mu} \partial_\mu \pi, \quad (19)$$

where as before, the first term gives a static effect and the second one dynamic couplings to the Goldstone. There are no bounds on F_2 and F_3 from static effects because when we set ϕ to its vev, \mathcal{L}_3 vanishes and \mathcal{L}_2 simply renormalizes the neutrino kinetic term. The dynamic couplings are

$$\mathcal{L}_2 = \frac{2M^2}{F_2^4} T \partial_0 \pi, \quad \mathcal{L}_3 = \frac{M^2}{2F_3^4} J^0 \partial^\mu \partial_\mu \pi. \quad (20)$$

In the case of gauged ghost condensation, we can covariantize (17) by replacing $\partial_\mu \phi$ with $D_\mu \phi$. This is a straightforward procedure and we do not give the details here.

III. REVIEW OF STATIC EFFECTS

We have seen how the vacuum expectation value for the ghost condensate gives rise to the static interactions in equations (14) and (19). In this section, we review how these terms generate Lorentz-violating dispersion relations for neutrinos and quote the strongest known bounds on sizes of these operators. Because these interactions may not be flavor-universal, we add flavor indices to the currents J^0 and T^{00} . The static Lorentz-violating Lagrangian is (i and j are flavor indices, not spatial indices)

$$\mathcal{L} = \bar{\mu}^{ij} J_{ij}^0 + \bar{a}^{ij} T_{ij}^{00}, \quad \bar{\mu} \equiv \frac{M^2}{F}, \quad \bar{a} \equiv \frac{M^4}{F_1^4}. \quad (21)$$

There is a subtlety in the definition of J^0 and T^{00} , especially in the presence of a Dirac mass. In the previous section, we defined J^0 and T^{00} to include left-handed projection operators, but there is no reason why right-handed neutrinos would not also feel the effect of Lorentz violation. However, to leading order in the neutrino mass matrix \bar{m}^2 and the Lorentz-violating parameters $\bar{\mu}$ and \bar{a} , we can ignore additional Lorentz-violating coupling to right-handed neutrinos. The effective Hamiltonian for left-handed neutrinos in the presence of equation (21) is

$$H_{ij} = \left(|p| \pm \mu + a|p| + \frac{m^2}{2|p|} \right) \delta_{ij} \pm \mu_{ij} + a_{ij}|p| + \frac{m_{ij}^2}{2|p|} + \dots \quad (22)$$

where the $+\mu$ is for left-handed neutrinos and the $-\mu$ is for right-handed anti-neutrinos. Here we use the notation that for a general $n \times n$ matrix \bar{X} ,

$$X = \frac{1}{n} \text{Tr } \bar{X}_{ij}, \quad X_{ij} = \bar{X}_{ij} - X \delta_{ij}. \quad (23)$$

Of course, when neutrinos travel in matter, one should also include the standard matter effect in (22) [25].

The flavor-universal terms m^2 , μ , and a affect neutrino kinematics, while the non-universal terms m_{ij}^2 , μ_{ij} , and a_{ij} dominantly affect neutrino oscillations. The effects from Lorentz-invariant neutrino masses are well studied and we will not elaborate on them further. In the following, we study flavor-universal and flavor-non-universal Lorentz-violating static effects.

A. Universal Terms

The universal Lorentz-violating terms affect the kinematics of the neutrinos, and one effect is to shift their group velocity away from the speed of light. For simplicity, we set the neutrino mass to zero and from (22) we find

$$v_g = 1 + a. \quad (24)$$

We confirm the known result that μ does not affect the group velocity. We see that the group velocity depends on a and is energy independent. While we do not consider them here, higher dimensional Lorentz-violating operators can yield energy dependent group velocities.

We can place bounds on a by considering experimental situations where the neutrino velocity is well measure over long time scales. At present, the best bounds come from supernova neutrinos. The only supernova which has been observed in neutrinos is SN1987A [26]. This supernova was at a distance of about 1.7×10^5 light years, which corresponds to travel time for the neutrinos of about

$$t_{87A} \sim 5 \times 10^{12} \text{ sec} \sim 10^{37} \text{ GeV}^{-1}. \quad (25)$$

The detected neutrino energies, of the order of a few tens of MeV, were in the expected range. The best bound we can get on the universal shift in the neutrino velocity can be derived by comparing the time of the neutrino signal to the light signal for SN1987A. The data indicates that the neutrino arrived within one day of the light, and therefore we have the bound [28]

$$|v_g - 1| \lesssim 10^{-8} \quad \Rightarrow \quad |a| \lesssim 10^{-8}. \quad (26)$$

Another consequence of modifying neutrino dispersion relations is that decay kinematics are modified. That is, the phase space factor for decays with neutrinos in the final state will be modified. In general, this can affect both the decay rates and the way these rates scale with the energy of the decaying particle. Such considerations were recently used in [29] in order to derive bounds from beta and pion decays. The former bound is stronger and gives a bound on the Lorentz-violating parameter μ :

$$\mu \lesssim 10^{-6} \text{ GeV}. \quad (27)$$

Note that [29] did not quote a bound on a from these consideration. Bounds on μ and a can also be derived from from comparing lifetime measurements at different energies. Since the Lorentz-violating term proportional to μ violates CPT, bounds on μ can be obtained by comparing lifetime measurements between particles that decay to neutrinos and anti-particles that decay to anti-neutrinos. While a detailed study of such bounds is needed, we do not expect the bounds to be much stronger than those we presented.

B. Non-universal Terms

Much stronger bounds can be derived on Lorentz-violating couplings that violate flavor because these couplings affect oscillation probabilities for neutrinos. Thus, solar, atmospheric and terrestrial neutrino experiments are all sensitive to these couplings.

Rough estimates of the bounds on Lorentz-violating couplings can be extracted by looking at the sensitivity for a given experiment to neutrino mass effects. That is, using equation (22), we can use the known sensitivity to $\Delta m^2/E$ as an estimate to the sensitivity to μ and

aE . From solar neutrino experiments and Kamland, which detect neutrinos with energy of order few MeV and are sensitive to $\Delta m^2 \sim 10^{-4} \text{ eV}^2$ we estimate for $i \neq e$

$$\mu_{ei} \lesssim \frac{\Delta m^2}{E} \sim 10^{-20} \text{ GeV}, \quad a_{ei} \lesssim \frac{\Delta m^2}{E^2} \sim 10^{-18}. \quad (28)$$

For atmospheric neutrinos a detailed study was done in [30, 31]. In a two generation approximation, it was found

$$\mu_{\mu\tau} \lesssim 10^{-22} \text{ GeV}, \quad a_{\mu\tau} \lesssim 10^{-24}. \quad (29)$$

We finally mention that bounds on non-universal Lorentz-violating couplings, $\mu_{ii} - \mu_{jj}$ and $a_{ii} - a_{jj}$ (for $i \neq j$) can be derived in a similar way. The numerical values of the bounds are the same order as magnitude as those derived above on off-diagonal couplings.

IV. DYNAMIC EFFECTS

In addition to static effects arising from the vev of the ghost condensate, we can derive complementary bounds on spontaneous Lorentz violation by studying fluctuations around the vev, namely the π field. As shown in section II A, π is a massless scalar field,³ and if it is coupled directly to the standard model we would expect to get constraints from fifth-force measurements as in the case of axions [32, 33]. In this paper, however, we restrict ourselves to couplings of the Goldstone to neutrinos, and because there is no way (yet) to assemble large coherent neutrino sources, we will rely on the novel effect of ether Čerenkov radiation to derive bounds on neutrino-Goldstone couplings. Further bounds from astrophysical considerations will be considered in [34]. As far as the neutrino-Goldstone interactions are concerned, bounds from star cooling arguments are much weaker than those we report here.

Neutrinos are relativistic and interact weakly with matter, so any interesting bounds on their coupling to the Goldstone field is likely to arise from processes that involve large distances and times. We consider two such dynamical effects in detail in the context of SN1987A and the CMBR. Using (4), the phase velocity of the Goldstone in the preferred rest frame of the universe is

$$v_\pi = \sqrt{\beta} \frac{k}{M}. \quad (30)$$

Therefore, a neutrino with any non-zero momenta will always be traveling faster than most k modes of the Goldstone field, making Čerenkov radiation from neutrinos into the Goldstone kinematically possible. As we will show, this has two important consequences. First, the emission of a single Goldstone quantum will deflect a relativistic particle by a large angle, enough to completely change its original path of travel. Second, the particle will lose energy,

³ A definition of massless is ambiguous in a Lorentz-violating theory. Here we use the term “massless field” to denote a field that has gapless excitations.

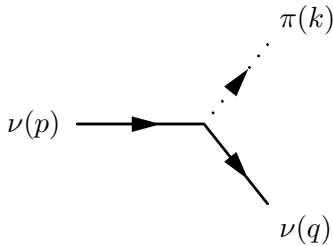


FIG. 1: The Feynman diagram for Goldstone-Čerenkov radiation from neutrinos.

which is a cumulative effect over several emissions. We use the first fact to study SN1987A where we have a handle on the number of neutrinos involved in a process, while we use the second fact to study the CMBR where we have knowledge about the energy stored in cosmological neutrinos.

The rate of energy loss due to Goldstone radiation from electrons was calculated in [13], where it was estimated that no observable effects could be seen from either Čerenkov radiation. Our case is quite different in that the calculation in [13] is non-relativistic and classical, whereas we do the calculation relativistically and in the quantum theory. Also, while the emission rate and the rate of energy loss are astronomically small, neutrinos compensate by traveling astronomical distances on astronomical time-scales.

The Čerenkov emission process we consider is $\nu \rightarrow \nu\pi$ as in figure 1. We take the initial and final neutrinos to be the same species, and thereby do not consider flavor changing vertices. This is justified as the experimental bounds on neutrino mixing are much stronger than the kind of bounds we can derive from our considerations. Furthermore, the main point in our analysis is that when a neutrino emits a Goldstone it is deflected from its original path of travel; whether it also changes flavor does not affect our results. We use (14) and (19) as the starting point of our calculation, the details of which we present in appendix B. Here we quote only the final results relevant for putting bounds on sizes of these operators. Neutrinos are in the ultra-relativistic kinematic regime and we only keep only the leading terms in an expansion in powers of the neutrino mass.

The quantities that are of greatest interest for experimental bounds are the emission rate Γ — the inverse of the average time for a neutrino of a given momentum p to emit a Goldstone quantum — and the rate of energy loss $-dE/dt$. We also list $\langle \cos \varphi \rangle$, which is the average deflection angle for a single emission event in order to show that it is large enough to completely change the trajectory of the emitting particle. All the results we quote should be taken as the leading order results in E/M where M is the scale of spontaneous Lorentz violation and E is the energy of the initial neutrino which does not change appreciably during a single emission. For the case of gauged ghost condensation we also work to leading order in the gauge coupling constant g . As already mentioned, all effects proportional to the neutrino mass are already suppressed in the relativistic regime, so we drop any subleading mass effects. Finally, for the purposes of this section, we can neglect any changes to the neutrino dispersion relation from the static effects in the previous section, as μ/E and a are

smaller than any other quantity appearing in our calculation.

For simplicity, we quote the results separately in different scenarios, which is justified since the operators do not interfere. We also substitute factors of F in favor of μ and a to simplify the comparison of static and dynamic effects. We begin by coupling the ghost condensate with the dimension-five operator of equation (14) for which we find

$$\Gamma = \frac{1}{4\pi\sqrt{\beta}} \frac{m^2}{M^3} \mu^2, \quad -\frac{dE}{dt} = \frac{1}{2\pi} \frac{m^2 E^2}{M^4} \mu^2, \quad \langle \cos \varphi \rangle = \frac{4E}{3M} \sqrt{\beta} \sim 0. \quad (31)$$

Note that these rates are suppressed by the neutrino mass. For this reason we also consider the effects of the dimension eight operators in equations (19), which, even though they are suppressed by higher powers of a high scale F , are not mass suppressed. We find

$$\Gamma = \frac{E^4}{12\pi\sqrt{\beta}M^3} a^2, \quad -\frac{dE}{dt} = \frac{E^6}{6\pi M^4} a^2, \quad \langle \cos \varphi \rangle = \frac{8E}{5M} \sqrt{\beta} \sim 0. \quad (32)$$

For the operators of (20) we find that \mathcal{L}_2 does not contribute, because it is proportional to the trace of the energy-momentum tensor which vanishes for massless on-shell neutrinos. For \mathcal{L}_3 , due to our definition of F_3 , the results can be obtained from (32) with the replacement $F_1 \rightarrow F_3$. (Note that this replacement has to be done in the definition of a , despite the fact that there is no static effect from \mathcal{L}_3 .)

For the coupling to the gauge ghost condensate, we work in the regime $g \gg m/E$ which, for relativistic neutrinos, holds for generic values of g . For simplicity, we also assume $g \gg E/M$ since in the opposite limit the theory coincides with ghost condensation. For the dimension five operator of equation (16) we find

$$\Gamma = \frac{g^3 \sqrt{\beta} E}{3\pi M^2} \mu^2, \quad -\frac{dE}{dt} = \frac{g^4 \beta E^2}{4\pi M^2} \mu^2, \quad \langle \cos \varphi \rangle = \frac{3}{5}. \quad (33)$$

As mentioned in section II C, these results are not mass suppressed as they were in the ungauged case, so the dimension eight operators in the gauged case should be subdominant. Yet, since the dimension five operators can be forbidden by a symmetry, we quote the result for the dimension eight operators as well:

$$\Gamma = \frac{gp^3}{15\pi\sqrt{\beta}M^2} a^2, \quad -\frac{dE}{dt} = \frac{g^2 p^4}{12\pi M^2} a^2, \quad \langle \cos \varphi \rangle = \frac{1}{7}. \quad (34)$$

Here, as for ghost condensation, \mathcal{L}_2 does not contribute and for \mathcal{L}_3 the results can be obtained from (34) with the replacement $F_1 \rightarrow F_3$.

We are now ready to use these results to constrain the sizes of the Lorentz-violating operators.

A. Bounds from SN1987A

First, we will apply our results to neutrinos arriving at the Earth from SN1987A. Since the observed number of neutrinos is consistent with existing supernovae models and we

found that if a neutrino radiates even a single Goldstone it will be thrown off its original trajectory, we demand

$$\Gamma t_{87A} \lesssim 1. \quad (35)$$

Note that our results for ghost condensation can be trusted as long as $E \ll M$. The observed neutrino spectrum from SN1987A implies that we need $M \gg 10 \text{ MeV}$ yet the bounds from gravitational experiments is $M \lesssim 10 \text{ MeV}$ [1]. Thus, our results in the case of ghost condensation should be taken as an order of magnitude estimate subject to $O(1)$ corrections. In the case of gauged ghost condensation, however, there is no difficulty taking $M \gg 10 \text{ MeV}$ [9] so this issue does not arise.

We adopt $M \sim 10 \text{ MeV}$ and a generic neutrino mass of $m \sim 0.1 \text{ eV}$ as reference values. We fix the supernova neutrino energy $E = 10 \text{ MeV}$. With this choice we obtain bounds on the dimension five operator in (31)

$$\mu \lesssim 10^{-11} \text{ GeV} \left(\frac{M}{10 \text{ MeV}} \right)^{3/2} \left(\frac{0.1 \text{ eV}}{m} \right), \quad (36)$$

and on the dimension eight operator in (32),

$$a \lesssim 10^{-17} \left(\frac{M}{10 \text{ MeV}} \right)^{3/2}. \quad (37)$$

For couplings to the gauged ghost condensate, we use a reference value $g \sim 10^{-3}$, and the bound from the dimension five operator gives

$$\mu \lesssim 10^{-15} \text{ GeV} \left(\frac{M}{10 \text{ MeV}} \right) \left(\frac{10^{-3}}{g} \right)^{3/2}. \quad (38)$$

Last, for the dimension eight coupling in gauged ghost condensation we get

$$a \lesssim 10^{-15} \left(\frac{M}{10 \text{ MeV}} \right) \left(\frac{10^{-3}}{g} \right)^{1/2}. \quad (39)$$

We could also use the fact that the neutrinos from SN1987A arrived with roughly their expected energy spectrum. That is, we could demand that $\Delta E \lesssim 10 \text{ MeV}$, but this would not improve the bounds already obtained. The reason is that $\Delta E/E = (T/E)(dE/dt)$ is generically smaller than ΓT by a factor of E/M (in ghost condensation) or g (in gauged ghost condensation) which are both assumed to be less than one in our analysis.

B. Bounds from Cosmology

Another way to probe Goldstone emissions is through cosmological observables, in particular the CMBR. In standard FRW cosmology, neutrinos decouple from the baryon/photon plasma around the time of nucleosynthesis. When the acoustic oscillations of the CMBR

are being formed, the neutrinos free-stream relativistically from over- to under-dense regions [35], and their energy density scales like radiation as the universe expands [36]. Non-standard interactions of neutrinos can affect the CMBR if they either inhibit free-streaming or if they transfer energy to a sector that redshifts differently than radiation. Generally, any theory that deviates from the standard picture can be tested with precision measurements of the microwave background. The energy density in neutrinos is roughly characterized by an effective number of neutrino species N_ν . Energy losses of neutrinos would appear as an effective number of species that is different from three. (Similar explorations are done in “late-time” neutrino models [37, 38].)

In ghost condensation, the interaction between the Goldstone and the neutrino is suppressed by a factor of $1/F$, so to a very good approximation the neutrinos are free-streaming over the time scales relevant for acoustic peak formation. However, the effect of overall energy transfer from neutrinos to Goldstones can be significant.

As we show in appendix C, in ghost condensation, the gravitational energy stored in Goldstone bosons redshifts like cold dark matter (CDM). Therefore, if a large number of Goldstones are created from neutrinos, the energy density in the universe that redshifts like radiation would be less than in the standard case. Such a difference would affect the CMBR observations made today. Of course, a detailed study of all cosmological observables is needed to fully understand the effect of Goldstone radiation. Below we only estimate bounds by demanding that neutrinos do not lose too much energy to Goldstones. That is, we demand that the total relative energy lost to Goldstones is not very large:

$$r \equiv - \int \frac{1}{E} \frac{dE}{dt} dt \lesssim 0.1. \quad (40)$$

Note that the time interval we are interested here — from neutrino decoupling to the formation of the CMBR — is roughly equal to the travel time of neutrinos from SN1987A to the earth. In the previous section, we demanded that not even a single Goldstone be emitted from SN1987A neutrinos. Here, we only demand that the emitted Goldstones do not drain too much energy from cosmological neutrinos, so one might expect that the bound from the CMBR would be weaker than from SN1987A. However, it is still interesting to study the CMBR, because if $M < 10$ MeV then all neutrino energies from SN1987A would be outside of the ghost condensate effective theory, and the bounds derived in the previous section could not be trusted. For the CMBR, though, the energy of the neutrinos redshifts down to roughly an eV towards the formation of the CMBR, and therefore we can still get some dynamic bounds for low values of M as well.

In order to perform the integral in (40), we have to know how the neutrino energy depends on time. Assuming a radiation dominated universe, we have [39]

$$t = C_1 T^{-2}, \quad C_1 \approx 3g_*^{-1/2} M_{\text{Pl}} \sim \text{MeV}^2 \text{ sec} \quad \Rightarrow \quad dt = -2C_1 T^{-3} dT. \quad (41)$$

The rate of energy loss can be written as

$$-\frac{dE}{dt} = C_2 E^n, \quad (42)$$

where C_2 and n depend on the specific operator we are considering. Taking an average neutrino energy of $E \sim 3T$ we get

$$r \sim 2 \times 3^{n-1} C_1 C_2 \int T^{n-4} dT \sim \frac{2 \times 3^{n-1} C_1 C_2}{n-3} T^{n-3} \Big|_{T_{max}}^{T_{min}}, \quad (43)$$

where the final step works for $n \neq 3$ and the integration limits are roughly from the time of neutrino decoupling $T_{max} = 1$ MeV to the formation of the CMBR $T_{min} = 1$ eV.

We can get a rough bound on the model parameters by using the requirement in (40). For the dimension five operator in (31), we have

$$C_2 = \frac{m^2 \mu^2}{2\pi M^4}, \quad n = 2, \quad \mu \lesssim 10^{-20} \text{ GeV} \left(\frac{M}{10 \text{ eV}} \right)^2 \left(\frac{0.1 \text{ eV}}{m} \right). \quad (44)$$

For the dimension eight operator in (32),

$$C_2 = \frac{a^2}{6\pi M^4}, \quad n = 6, \quad a \lesssim 10^{-10} \left(\frac{M}{10 \text{ MeV}} \right)^2. \quad (45)$$

As we show in appendix C, the equation of state for gauged ghost condensation is that of radiation. Therefore one may be able to obtain only weak bounds in this case because the equation of state for neutrinos and the longitudinal mode are identical to leading order. On the level of the rough bounds with which we are concerned, we cannot place additional bounds on gauged ghost condensation through cosmological considerations.

V. DISCUSSION AND CONCLUSIONS

All experimental evidence to date confirms Lorentz invariance to be a highly accurate symmetry of nature. Of course, Lorentz invariance could be violated, and the way it is violated would have definite experimental implications. In this paper, we have focused on the possibility that general relativity correctly describes nature at all energies below M_{Pl} , but that both diffeomorphisms and Lorentz invariance are spontaneously broken at some scale M . However, it is possible that general relativity is only one limit of the true theory of gravity, in which case there are two additional possibilities. First, Lorentz invariance could be an accidental symmetry of elementary particle interactions that is simply absent at higher energies. Second, just as Galilean invariance is the small velocity limit of Lorentz invariance, Lorentz invariance could be just be some limit of a more fundamental symmetry.

Spontaneous Lorentz violation allows us to make definite experimental predictions in the low energy effective theory below the scale M without having to postulate a full theory of

modified gravity in the ultraviolet. In particular, by assuming that general relativity holds, the existence of Lorentz-violating vevs and Goldstone bosons are robust, and they lead to novel phenomena that can be used to probe spontaneous Lorentz violation.

In this paper, we assumed that the Lorentz-violating sector coupled only to neutrinos, and we studied how neutrinos can place bounds on the specific models of ghost condensation and gauged ghost condensation. Static effects arose from the presence of a Lorentz-violating vev. The dominant static effect is a modification of the neutrino dispersion relation, which has been extensively studied in the literature. Here, we focused on dynamic effects from spontaneous Lorentz violation. Like any spontaneously broken symmetry, spontaneously broken Lorentz symmetry is associated with a massless Goldstone boson. Every static interaction is accompanied by a novel neutrino-Goldstone coupling. We have seen that when neutrinos travel in vacuum they can lose energy and change their direction of motion due to Goldstone emission. This Goldstone-Čerenkov effect can be used to probe the mechanism of spontaneous Lorentz violation.

We identified two kinds of observables that can be used to probe this dynamic effect. First, we looked at data on neutrinos from astrophysical sources, in particular, from SN1987A. If neutrinos from SN1987A emitted Goldstone bosons on their way to the Earth, they would have deflected away and the number of neutrinos to arrive at the Earth would have changed. Because the data tells us that the neutrinos travel from the supernova basically without interaction, we can put stringent constraints on the size of Lorentz violation. We also applied cosmological considerations to put bounds on the model parameters. We know that the CMBR power spectrum agrees with the presence of three standard neutrino species. Significant energy transfer from neutrinos into the Goldstone field would affect standard cosmology, and would therefore be inconsistent with observations.

We summarize the bounds on the coupling of neutrinos to the Lorentz-violating sector in Table I. We quote bounds on two kind of operators, the dimension five operator in (13) and the dimension eight operator in (17). We quote bounds from static and dynamic effects. The main conclusion of our paper can be read off the table. We see that dynamic effects can be much more effective in probing Lorentz violation than static effects, especially for flavor-universal couplings. That is, ignoring the presence of the Goldstone boson is generically not justified.

Future experiments can be used to further probe Lorentz violation, enabling us to better constrain the degree of Lorentz violation or to discover it. Detecting neutrinos from cosmological distances, as in gamma ray bursts, will provide much stronger probes as such neutrinos travel very long distances. A close by supernova observed with a high statistics neutrino signal would also be very useful to refine our rough bounds. Finally, the study of the cosmological implication of the neutrino-Goldstone interaction has to be refined. Here, we gave a very rough estimate of the effect using only energy-loss arguments. Clearly, a detailed study will be useful to fully understand how such effects can be discovered or further bounded.

TABLE I: Summary of bounds on Lorentz-violating couplings. In all cases only rough estimates are given. The first group corresponds to bounds from static effects on non-universal (first two entries) and universal (third and fourth entries) Lorentz-violating couplings. In the second group bounds on the universal coupling are given both for ghost condensation (fifth and sixth entries) and gauged ghost condensation (last entry). Here, M is the scale of spontaneous Lorentz violation, m is the neutrino mass, and g is the gauge coupling in gauged ghost condensation. N/A is given when no bounds can be obtained or when bounds are unavailable but are expected not to be significant. See the text for more details.

	μ [GeV]	a [number]
Static bounds		
Atmospheric (non-universal)	10^{-22}	10^{-24}
Solar/Kamland (non-universal)	10^{-20}	10^{-18}
SN1987A	N/A	10^{-8}
Decay kinematics	10^{-6}	N/A
Dynamic bounds		
SN1987A (ghost condensation)	$10^{-11} \left[\frac{M}{10 \text{ MeV}} \right]^{3/2} \left[\frac{0.1 \text{ eV}}{m} \right]$	$10^{-17} \left[\frac{M}{10 \text{ MeV}} \right]^{3/2}$
CMBR (ghost condensation)	$10^{-22} \left[\frac{M}{1 \text{ eV}} \right]^2 \left[\frac{0.1 \text{ eV}}{m} \right]$	$10^{-11} \left[\frac{M}{1 \text{ MeV}} \right]^2$
SN1987A (gauged ghost condensation)	$10^{-15} \left[\frac{M}{10 \text{ MeV}} \right] \left[\frac{10^{-3}}{g} \right]^{3/2}$	$10^{-15} \left[\frac{M}{10 \text{ MeV}} \right] \left[\frac{10^{-3}}{g} \right]^{1/2}$

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APPENDIX A: GAUGED GHOST CONDENSATION

Here we give some details of gauged ghost condensation, following [9]. Starting with the Lagrangian in equation (8), we derive the dispersion relation and polarization of the Goldstone mode in gauged ghost condensation. For ease of discussion, we work out the details in the limit $k \ll gM$, and then quote results in the general case to see how ghost condensation is recovered from gauged ghost condensation in the $g \rightarrow 0$ limit. The gauged

ghost Lagrangian is:

$$\mathcal{L} = -\frac{1}{4g^2}F_{\mu\nu}^2 + \frac{1}{8M^4}\left((D_\mu\phi)^2 - M^4\right)^2 - \frac{\beta}{2M^2}(\partial^\mu D_\mu\phi)^2 - \frac{\beta'}{2M^2}(\partial_\mu D_\nu\phi)^2 + \dots \quad (\text{A1})$$

First, we can go to a “unitary” gauge where $\phi \equiv M^2 t$. Keeping only terms quadratic in A_μ we have

$$\mathcal{L} = -\frac{1}{4g^2}F_{\mu\nu}^2 + \frac{M^2}{2}A_0^2 - \frac{\beta}{2}(\partial^\mu A_\mu)^2 - \frac{\beta'}{2}(\partial_\mu A_\nu)^2 + \dots, \quad (\text{A2})$$

We can absorb β' into a redefinition of g and β , so we set $\beta' = 0$. Since the mode A_0 has mass gM we can integrate it out. Ignoring k/M corrections to the dispersion relations of the massless polarizations, we can simply set $A_0 = 0$, and the A_i Lagrangian is

$$\mathcal{L} = \frac{1}{2g^2}(\partial_0 A_i)^2 - \frac{1}{2g^2}(\partial_i A_j)^2 + \frac{1}{2g^2}(1 - \beta g^2)(\partial^i A_i)^2. \quad (\text{A3})$$

Expanding A_i in plane waves with $k_\mu = (E, 0, 0, k)$, the classical polarization vectors are as follows. The transverse modes have Lorentz invariant dispersion relations and are

$$E^2 = k^2; \quad \epsilon_i^1 = (g, 0, 0), \quad \epsilon_i^2 = (0, g, 0). \quad (\text{A4})$$

The longitudinal mode, however, has a Lorentz-violating dispersion relation

$$E^2 = \beta g^2 k^2; \quad \epsilon_i^L = (0, 0, g) = g k^i / |k|. \quad (\text{A5})$$

To verify the normalization of the polarizations, we calculate the vector field propagator

$$\langle A_i A_j \rangle = \frac{g^2}{E^2 - k^2} \left(\delta_{ij} - (1 - \beta g^2) \frac{k_i k_j}{E^2 - \beta g^2 k^2} \right). \quad (\text{A6})$$

As expected from the cutting rules, we find

$$\langle A_i A_j \rangle (E^2 - k^2) \Big|_{E=k} = \sum_{n=1,2} \epsilon_i^n \epsilon_j^{n*}, \quad \langle A_i A_j \rangle (E^2 - \beta g^2 k^2) \Big|_{E=\sqrt{\beta} g k} = \epsilon_i^L \epsilon_j^{L*}. \quad (\text{A7})$$

While not directly relevant to our discussion, it is instructive to consider the more general case with generic values of gM and k . This will allow us to see how original ghost condensation is recovered in the $g \rightarrow 0$ limit. In that case, we need to work with the full A_μ field and cannot integrate out A_0 . Using equation (A2) with $\beta' = 0$, there are still two transverse polarization vectors with relativistic dispersion relations:

$$E^2 = k^2; \quad \epsilon_\mu^1 = (0, g, 0, 0), \quad \epsilon_\mu^2 = (0, 0, g, 0). \quad (\text{A8})$$

There are also two “longitudinal” modes. The first is a ghost excitation (*i.e.* the residue at the relevant pole of the propagator is negative), but it has a massive dispersion relation

$$E_{\text{ghost}}^2 = \frac{M^2}{\beta} + (2 - g^2 \beta) k^2 + \mathcal{O}(k^4/M^2), \quad (\text{A9})$$

so for $E \ll M/\sqrt{\beta}$, this unhealthy mode is never excited. The Goldstone longitudinal mode has dispersion relation

$$E_L^2 = \beta g^2 k^2 + \beta(1 - g^2 \beta)^2 \frac{k^4}{M^2} + \mathcal{O}(k^6/M^4), \quad (\text{A10})$$

which matches the ghost condensate dispersion relation in the $g \rightarrow 0$ limit. The normalized polarization vector is

$$\epsilon_\mu^L = \left(\frac{E}{gM^2} \left((1 - g^2 \beta)k + \mathcal{O}(k^2/M^2) \right), 0, 0, g + \mathcal{O}(k^2/M^2) \right), \quad (\text{A11})$$

which indeed reproduces equation (A5) up to $\mathcal{O}(k^2/M^2)$ corrections. However, we see that the zero gauge coupling limit is singular if we do an expansion in k/M and then try to take $g \rightarrow 0$. This is to be expected, because with $g = 0$ the Goldstone energy scales as k^2 , but for finite g , $E \sim k$. Expanding in g first, and then considering (k/M) corrections we get

$$\epsilon_\mu^L = \left(\frac{E}{M} \left(1 + \mathcal{O}(k^2/M^2) \right) + \mathcal{O}(g^2), 0, 0, \frac{k}{M} + \mathcal{O}(k^2/M^2) + \mathcal{O}(g^2) \right). \quad (\text{A12})$$

And we see that at the level of longitudinal polarizations

$$D_\mu \phi \equiv M^2 \delta_\mu^0 + M A_\mu^L \quad \Rightarrow \quad \partial_\mu \phi = M^2 \delta_\mu^0 + \partial_\mu \pi, \quad (\text{A13})$$

in the $g \rightarrow 0$ limit.

APPENDIX B: GOLDSTONE-ČERENKOV CALCULATION

In this appendix, we give details of our dynamic calculations. We consider an ultra-relativistic (in the ether rest frame) neutrino with mass m that emits a Goldstone

$$\nu(p) \rightarrow \nu(q) + \pi(k), \quad (\text{B1})$$

as in figure 1. We consider the case where the initial and final neutrino is the same. We use

$$p_\mu = (E_{in}, 0, 0, p), \quad q_\mu = (E_{out}, q \sin \varphi, 0, q \cos \varphi), \quad k_\mu = (E_\pi, k \sin \theta, 0, k \cos \theta). \quad (\text{B2})$$

We define for a general four vector V_μ , $\tilde{V}_\mu = (V, -\vec{V})$. In the following, we work to leading order in m/E . Then, $E_{in} = p$ and $E_{out} = q$. We express all constrained kinematic variables in terms of p and θ .

The differential rate is given by the standard formula [40]

$$d\Gamma = \frac{d^3 k}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \frac{1}{(2E_{in})(2E_{out})(2E_\pi)} |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p - q - k). \quad (\text{B3})$$

We always work in regions where the effective theory is valid. Then, $E_\pi \ll k$ and to leading order in E_π/k we have $p = q$ [see (B7) and (B19)]. It turns out to be more convenient to

use the $\delta^{(3)}$ function to eliminate \vec{q} . Performing the trivial integration over the azimuthal angle we have an expression that depends only on p , k and $\cos \theta$

$$d\Gamma = \frac{k^2 dk d(\cos \theta)}{16\pi p^2 E_\pi(k)} |\mathcal{M}|^2 \frac{\delta(k - k_{on})}{f'(k_{on})}. \quad (\text{B4})$$

Here,

$$f(k) = E_\pi(k) + E_{out}(k) - E_{in}(k), \quad (\text{B5})$$

is the energy conservation condition where the solution of the on-shell condition is denoted by k_{on} . We also used the fact that when $f(x) = 0$ has a single solution $x = x_0$ then

$$\int \dots \delta(f(x)) dx = \int \dots \frac{\delta(x - x_0)}{|f'(x_0)|} dx. \quad (\text{B6})$$

In order to calculate the rate we need to know $f(k)$ (that is, the dispersion relation of the Goldstone) and the matrix element, \mathcal{M} . Both are model dependent, and below we continue the calculations in the different models we are considering.

1. Ghost Condensation

We use the Goldstone dispersion relation from (4). We work in the limit where the effective theory is valid, that is, when $k/M \ll 1$. While we calculate to subleading order in k/M , we report only the leading order results.

The energy and momentum conservation equations for ghost condensation are

$$p = q + \sqrt{\beta} \frac{k^2}{M}, \quad q^2 = p^2 + k^2 - 2pk \cos \theta. \quad (\text{B7})$$

The solutions to first order in k/M are

$$q = p, \quad k_{on} = 2p \cos \theta, \quad \cos \varphi = -\cos 2\theta, \quad f'(k_{on}) = \cos \theta, \quad (\text{B8})$$

and the allowed values for $\cos \theta$ are $0 \leq \cos \theta \leq 1$. We then get from (B4)

$$d\Gamma = \frac{M}{16\pi p^2 \sqrt{\beta} \cos \theta} |\mathcal{M}|^2 d(\cos \theta). \quad (\text{B9})$$

To continue further, we need to specify the interaction. We start with the dimension five operator in (14). This operator leads to a $\nu - \nu - \pi$ vertex that is proportional to \not{k}/F . Thus, for the amplitude we get

$$\mathcal{M} = \frac{1}{F} \bar{u}(q) \not{k} P_L u(p) = \frac{m}{F} \bar{u}_q \gamma^5 u_p, \quad (\text{B10})$$

where $P_L = (1 - \gamma^5)/2$ and the equation of motion of the fermion was used in the last step. (While static effects modify the neutrino dispersion relation, this change is a higher order

effect and we verify that it is indeed negligible when considering the neutrino kinematics.) Note that the amplitude is proportional to the neutrino mass. This can be understood by the fact that for a massless neutrino, the dimension five interaction is not physical because it can be rotated away by a field redefinition. Using standard spin sum and trace technology we get

$$\sum |\mathcal{M}|^2 = \frac{m^2}{F^2} \text{Tr}(\not{p}\gamma^5 \not{q}\gamma^5) = -\frac{4m^2}{F^2} p_\mu q^\mu = \frac{2m^2 k^2}{F^2} = \frac{8m^2 p^2 \cos^2 \theta}{F^2}, \quad (\text{B11})$$

where in the last step we use the on-shell condition, (B8). While we do not present here the polarized results explicitly, we note that the interaction requires a spin flip.

Using (B11) in (B9) we can calculate the observables we are after. In particular, we calculate the total width, Γ , the average energy lost to the Goldstone, $-dE/dt$, and the average deflection of the neutrino, $\langle \cos \varphi \rangle$. To leading order in p/M we get

$$\begin{aligned} \Gamma &= \int d\Gamma = \frac{Mm^2}{2\pi\sqrt{\beta}F^2} \int_0^1 \cos \theta d(\cos \theta) = \frac{Mm^2}{4\pi\sqrt{\beta}F^2}, \\ -\frac{dE}{dt} &= \int d\Gamma E_k = \frac{m^2 p^2}{2\pi F^2}, \\ \langle \cos \varphi \rangle &= \frac{\int d\Gamma \cos \varphi}{\int d\Gamma} = \frac{4\sqrt{\beta}p}{3M} \sim 0. \end{aligned} \quad (\text{B12})$$

Next we repeat the calculation using the dimension eight operators. In that case, we set the neutrino mass to zero, since, unlike the dimension five case, it is not required in order to have a nonzero effect. We start with \mathcal{L}_1 in (19), where the $\nu - \nu - \pi$ vertex is

$$\frac{M^2}{F_1^4} (\gamma^0 k_\mu p^\mu + \not{k} p^0 - 2k^0 \not{p}) P_L. \quad (\text{B13})$$

The last two terms vanish for on-shell neutrinos and thus the amplitude is

$$\mathcal{M} = \frac{M^2}{F_1^4} \bar{u}(q) \gamma^0 k_\mu p^\mu P_L u(p). \quad (\text{B14})$$

Note that this amplitude conserves spin. Summing over spins we get

$$\sum |\mathcal{M}|^2 = \frac{M^4}{F_1^8} (k_\mu p^\mu)^2 \text{Tr}(\not{q}\gamma^0 P_L \not{p}\gamma^0) = \frac{2M^4}{F_1^8} (k_\mu p^\mu)^2 (p_\nu \tilde{q}^\nu) = \frac{16M^4}{F_1^8} p^6 \cos^4 \theta \sin^2 \theta, \quad (\text{B15})$$

where in the last step we used the on-shell condition. Using (B15) in (B9) we get to leading order in p/M

$$\Gamma = \frac{M^5 p^4}{12\pi\sqrt{\beta}F_1^8}, \quad -\frac{dE}{dt} = \frac{M^4 p^6}{6\pi F_1^8}, \quad \langle \cos \varphi \rangle = \frac{8\sqrt{\beta}p}{5M} \sim 0. \quad (\text{B16})$$

The calculation for the other dimension eight operators is very similar. In the massless limit \mathcal{L}_2 in (17) gives no effects for on-shell neutrinos because it is proportional to the trace

of the energy momentum tensor. For \mathcal{L}_3 , we go through essentially the same analysis as for \mathcal{L}_1 and find

$$|\mathcal{M}|^2 = \frac{16M^4}{F_3^8} p^6 \cos^4 \theta \sin^2 \theta, \quad (\text{B17})$$

which leads to the same result as in (B16) with the replacement $F_1 \rightarrow F_3$. Note that there is no fundamental reason that \mathcal{L}_1 and \mathcal{L}_3 give the same results.

2. Gauged Ghost Condensation

We move to perform the calculation in the gauged case. The process we are after is the same as for the global case but in the gauged case the emitted boson is a vector. The neutrino can emit the longitudinal mode of the vector since that is the one that has a Lorentz-violating dispersion relation. The dispersion relation of the longitudinal mode is given in equation (10). We assume

$$g \gg \frac{k}{M}, \quad g\sqrt{\beta} \gg \frac{m}{p}, \quad g\sqrt{\beta} \ll 1, \quad (\text{B18})$$

and keep only leading contributions in these small quantities.

The energy and momentum conservations for the gauged case are

$$p = q + \sqrt{\beta} g k, \quad q^2 = p^2 + k^2 - 2pk \cos \theta. \quad (\text{B19})$$

The solutions to first order in the small quantities is the same as for ghost condensation (B9) (differences arises in high order terms). Going through the same steps as in the previous subsection, we derive from (B4)

$$d\Gamma = \frac{1}{8\pi p g \sqrt{\beta}} |\mathcal{M}|^2 d(\cos \theta). \quad (\text{B20})$$

The vertex can be derived from (16) by substituting the correctly normalized polarization vector for the longitudinal mode given in (11) and it is proportional to $\not{p}M/F$. For the amplitude we then get

$$\mathcal{M} = \frac{M}{F} \bar{u}(q) \not{p} P_L u(p) = \frac{M}{F} g \bar{u}(q) \hat{k} \cdot \vec{\gamma} P_L u(p) = -\frac{g^2 \sqrt{\beta} M}{F} \bar{u}(q) \gamma^0 P_L u(p), \quad (\text{B21})$$

where in the last step we use the fact that in the massless limit $\bar{u}(q) \not{p} P_L u(p) = 0$. Note that unlike ghost condensation the rate is not proportional to the neutrino mass. This is to be expected as in the gauged case the dimension five interaction cannot be rotated away in the massless limit. Summing over spins we get

$$|\mathcal{M}|^2 = \frac{g^4 \beta M^2}{F^2} \text{Tr}(\not{q} \gamma^0 P_L \not{p} \gamma^0) = \frac{2g^4 \beta M^2}{F^2} (p_\mu \tilde{q}^\mu) = \frac{4g^4 \beta M^2}{F^2} p^2 \sin^2 \theta, \quad (\text{B22})$$

where we used the on-shell condition in the last step. Using (B22) in (B20) we can calculate the relevant observables and we get

$$\Gamma = \frac{g^3 \sqrt{\beta} M^2 p}{3\pi F^2}, \quad -\frac{dE}{dt} = \frac{g^4 \beta M^2 p^2}{4\pi F^2}, \quad \langle \cos \varphi \rangle = \frac{3}{5}. \quad (\text{B23})$$

Next we consider the dimension eight operators in the gauged case. The vertex we get from covariantizing the operator in equation (17) similar to (16) and using the polarization vector for the longitudinal mode we find the amplitude

$$\mathcal{M} = \frac{gM^3}{F_1^4} \bar{u}(q) \left(-\gamma^0 \vec{p} \cdot \hat{k} + p^0 \hat{k} \cdot \vec{\gamma} \right) P_L u(p) = -\frac{M^3 g}{F_1^4 k} p_\mu \tilde{k}^\mu \bar{u}(q) \gamma^0 P_L u(p), \quad (\text{B24})$$

where in the last step we trade $\vec{k} \cdot \vec{\gamma}$ for $-k^0 \gamma^0$. Using a spin sum we get

$$|\mathcal{M}|^2 = \frac{M^6 g^2}{k^2 F_1^8} (p_\mu \tilde{k}^\mu)^2 \text{Tr} \left(\not{q} \gamma^0 P_L \not{p} \gamma^0 \right) = \frac{2M^6 g^2}{k^2 F_1^8} (p_\mu \tilde{k}^\mu)^2 (p_\nu \tilde{q}^\nu) = \frac{4M^6 g^2}{F_1^8} p^4 \sin^2 \theta \cos^2 \theta, \quad (\text{B25})$$

where we used the on-shell condition in the last step. Using the above in (B20) we find

$$\Gamma = \frac{M^6 g p^3}{15\pi \sqrt{\beta} F_1^8}, \quad -\frac{dE}{dt} = \frac{M^6 g^2 p^4}{12\pi F_1^8}, \quad \langle \cos \varphi \rangle = \frac{1}{7}. \quad (\text{B26})$$

Finally, we perform the calculation for the remaining dimension eight operators. As in ghost condensation, \mathcal{L}_2 does not contribute for on-shell neutrinos. For \mathcal{L}_3 we found that the results are the same as for \mathcal{L}_1 up to the replacement $F_1 \rightarrow F_3$.

APPENDIX C: PARTICLE PHYSICS VS. GRAVITATIONAL ENERGY

In section IV B, we used an energy-loss argument to bound the amount of energy transferred from neutrinos to Goldstone bosons prior to the formation of the CMBR. The key to this argument is that neutrinos and Goldstone bosons redshift differently. In particular, in the appropriate approximation, neutrinos redshift like radiation and Goldstone bosons like CDM. In this appendix we verify this, while pointing out an interesting subtlety in calculating the equation of state for the Goldstone boson.

The usual way [39] to figure out the equation of state for a particle is to note that in a FRW background, any energy-momentum tensor has the form of a perfect fluid. Thus, conservation of stress-energy provides the first law of thermodynamics

$$d(\rho a^3) = -\mathcal{P} d(a^3), \quad (\text{C1})$$

where ρ is the density and \mathcal{P} the pressure. This law yields the physical intuition that energy in a comoving volume is equal to minus the pressure times the same volume. Assuming the simple equation of state $\mathcal{P} = w\rho$, the first law thermodynamics gives the evolution of the energy density as

$$\rho \sim a^{-3(1+w)}. \quad (\text{C2})$$

In general, for a normal species particle whose dispersion relation goes as $E \sim k^n$, the corresponding to the equation of state is

$$w = \frac{n}{3}. \quad (\text{C3})$$

For cold dark matter where there is no energy redshift, $w = 0$, and for relativistic neutrinos where $E \sim k$, $w = 1/3$. One might then conclude that for the Goldstone boson where $E \sim k^2$, the equation of state would be $w = 2/3$.

However, this argument assumes that the equivalence principle is observed. In ghost condensation, however, time diffeomorphisms are spontaneously broken, so there is no reason to expect that the current associated with space-time translations (the “particle physics” energy) would be the same as the stress-energy tensor (the gravitational energy) [1]. To leading order in k/M , the stress energy tensor for the ghost condensate is

$$T_{\mu\nu} = \frac{1}{2M^4} \left((\partial_\mu \phi)^2 - M^4 \right) \partial_\mu \phi \partial_\nu \phi - \frac{2\beta}{M^2} \partial_\mu \partial_\nu \phi (\partial^\alpha \partial_\alpha \phi) - g_{\mu\nu} \mathcal{L}, \quad (\text{C4})$$

and expanding around the vacuum, $\partial_\mu \phi = M^2 \delta_\mu^0 + \partial_\mu \pi$, the gravitational energy density for Goldstone is

$$\rho_{\text{grav}} = T_{00} = M^2 \dot{\pi}^2 + 2\dot{\pi}^2 - \frac{1}{2} (\nabla \pi)^2 + \frac{\beta}{2M^2} (\nabla^2 \pi)^2 + \dots \quad (\text{C5})$$

which looks nothing like the particle physics energy density for the Goldstone:

$$\rho_{\text{particle}} = \frac{1}{2} \dot{\pi}^2 + \frac{\beta}{2M^2} (\nabla^2 \pi)^2. \quad (\text{C6})$$

As for the pressure of the Goldstone, we find

$$3\mathcal{P} = \sum_i T_{ii} = \frac{3}{2} \dot{\pi}^2 - \frac{3\beta}{2M^2} (\partial_\alpha \partial^\alpha \pi)^2 + \frac{2\beta}{M^2} (\nabla^2 \pi \partial_\alpha \partial^\alpha \pi) + \dots \quad (\text{C7})$$

Now as we impose the Goldstone equation of motion and average over time, the $\dot{\pi}$ term in equation (C4) averages to zero and several terms cancel each other, leaving

$$\rho_{\text{grav}} = -\frac{1}{2} (\nabla \pi)^2 + \dots \quad \mathcal{P} = -\frac{2\beta}{3M^2} (\nabla^2 \pi)^2 + \dots \quad (\text{C8})$$

which leads to the equation of state

$$w_{\text{Goldstone}} = \frac{4\beta}{3} \frac{k^2}{M^2} \sim 0. \quad (\text{C9})$$

There is another, more intuitive way to figure out the equation of state for the Goldstone boson using equation (C4). A relativistically normalized plane wave of π in a box of volume V takes the form

$$\pi(x, t) = \frac{1}{\sqrt{EV}} e^{iEt - i\vec{k} \cdot \vec{x}}, \quad (\text{C10})$$

where $E = \sqrt{\beta}k^2/M$. Plugging into T_{00} and averaging over some reasonable amount of time we have

$$T_{00} = \frac{1}{EV} \left(\frac{5}{2}E^2 - \frac{1}{2}k^2 \right) = -\frac{M}{2\sqrt{\beta}V} + \mathcal{O}(k/M). \quad (\text{C11})$$

We see that T_{00} for a Goldstone plane wave is the same as for a non-relativistic particle plane wave with negative gravitational mass $-M/\sqrt{\beta}$.⁴

As the universe expands, the only redshift in T_{00} comes from the volume factor, so we recover (C9). Thus performing the analysis both ways leads us to the same conclusion, namely that indeed the equations of state of neutrino and the Goldstone boson differ. Despite having an $E \sim k^2$ dispersion relation, the gravitational energy in Goldstone radiation redshifts like cold dark matter with negative mass.

We can go through a similar analysis in the case of gauged ghost condensation. In that case, we expect the energy carried by the longitudinal mode to scale like radiation. The reason is that in gauged ghost condensation the gravitational and particle energies are the same up to small corrections [9]. Below we show that this is indeed the case.

The energy momentum tensor for the gauged case is given by

$$T^{\mu\nu} = -\frac{1}{g^2}F^{\mu\alpha}F_{\alpha}^{\nu} + \frac{1}{2M^4}D^{\mu}\phi D^{\nu}\phi \left((D\phi)^2 - M^4 \right) - \frac{2\beta}{M^2}\partial^{\mu}D^{\nu}\phi (\partial^{\alpha}D_{\alpha}\phi) - g^{\mu\nu}\mathcal{L}. \quad (\text{C12})$$

Now we go to the unitary gauge where $D_{\mu}\phi = \delta_{\mu 0}M^2 + MA_{\mu}$ and we work in the $g \gg k/M$ limit where the A_0 mode is heavy and can be integrated out by setting it to its equation of motion. To leading order this has the effect of setting the contribution from the $((D\phi)^2 - M^4)$ to zero so we find

$$T^{00} = \frac{1}{2g^2} \left(\sum_i (F^{0i})^2 + \sum_{i>j} (F^{ij})^2 \right) + \frac{\beta}{2} (\vec{\nabla} \cdot \vec{A})^2 + \dots \quad (\text{C13})$$

where A_0 has been set to zero to leading order by its equation of motion. We also find

$$\sum_i T^{ii} = T^{00} \quad (\text{C14})$$

so that

$$w_{\text{gauged ghost}} = \frac{1}{3}, \quad (\text{C15})$$

and the gauged ghost redshifts like radiation. Note that the result we have found includes the transverse modes also, which one would intuitively expect to redshift like radiation. The main point is that all modes — in particular the longitudinal mode which has a Lorentz-violating dispersion relation — have the equation of state characteristic of radiation.

⁴ Since we are dealing with a theory with broken time diffeomorphisms, one should not be concerned by the presence of an anti-gravitating field. For the rough estimates presented here, all we care about is that the gravitational energy carried by Goldstones differs from neutrinos. A complete analysis of CMBR signals of Lorentz violation would require a full understanding of the gravitational dynamics of the Goldstone.

In fact, we can show this another way, by plugging the polarization vector for the longitudinal mode into (C13) to isolate its contribution to the gravitational energy density. The relativistically normalized plane wave is

$$\vec{A}_L = \frac{1}{\sqrt{EV}} \vec{\epsilon}_L e^{iEt - i\vec{k}\cdot\vec{x}}. \quad (\text{C16})$$

We find to leading order that

$$\rho_{\text{grav}} = \frac{g^2 k^2 \beta}{EV} = \frac{gk\sqrt{\beta}}{V}, \quad (\text{C17})$$

so the longitudinal mode redshifts like radiation and has the equation of state of (C15).

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